Name: _____   Practice Test Chapter 5

NO CALCULATORS

Find each of the following integrals. Show all substitutions used(when necessary).

1. $\int x^2(3+x)^2\,dx$

2. $\int \left(\dfrac{1}{x^2} - \sin x\right) dx$

3. $\int 6x^2\sqrt{3x^3-1}\,dx$

4. $\int \sqrt{\cos(6x)}\,\sin(6x)\,dx$

5. $\int \dfrac{dx}{3x-1}$

6. $\int \dfrac{x\,dx}{\sqrt{x^2-5}}$

Evaluate the following. Show all work.

7. $\displaystyle\int_{0}^{\frac{\pi}{4}}(4x+\sec^2 x)\,dx$

8. $\int_{0}^{2}(e^x - 4x)\,dx$

9. Suppose that f and g are continuous functions and that
$$\int_{2}^{5} f(x)dx = 13,\ \int_{2}^{5} g(x)dx = 9,\ \int_{0}^{2} f(x)dx = -5,\ \text{and}\ \int_{2}^{8} f(x)dx = 20$$

Find each of the following:

a. $\displaystyle\int_{0}^{5} f(x)dx$

b. $\displaystyle\int_{2}^{5}[3f(x)-g(x)]dx$

c. $\displaystyle\int_{8}^{5} f(x)dx$

d. $\displaystyle\int_{2}^{5}[3f(x)-4]dx$

10. Given that $\dfrac{dy}{dx}=x^3+1$. Find the equation for y if $y(2)=7$.

11. Given the graph of $f'(x)$ shown at the right

Given $f(0) = 3$, find each of the following:
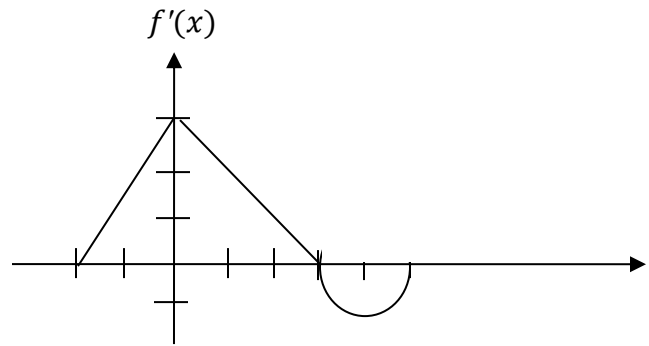
$f'(x)$

a) $f(3)$

b) $f(-2)$

c) $f'(2)$

d) $f''(-1)$

e) Interval(s) where $f(x)$ is decreasing

f) Value(s) of x where $f(x)$ has a relative maximum or minimum. Justify your answer.

g) Interval(s) where $f(x)$ is concave up

h) Value(s) of x where $f(x)$ has an inflection point. Justify your answer.

CALCULATORS ALLOWED

12. Find the area under the curve $y = 2^x$ from $x = 0$ to $x = 4$ using the following approximations. For each be sure to show your graph, set-up, and intermediate calculations!! Let $n = 4$.
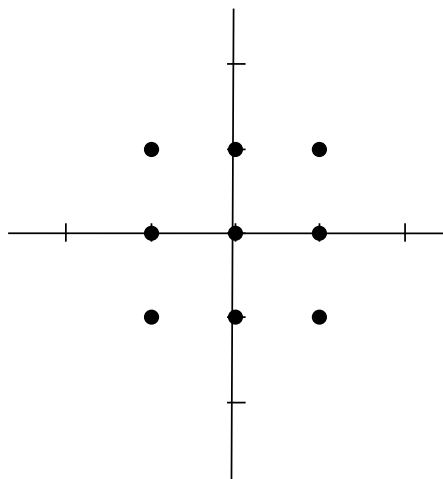
   a) Right endpoint rectangles.   b) Trapezoids

13. Estimate $\int_0^{90} f(x)$ by using a trapezoidal approximation with 6 equal subdivisions.

x	0	15	30	45	60	75	90
f(x)	12	11	10	10	8	7	0

14. Evaluate $\int_2^5 \ln(x)(\cos 2x)dx$

15. Draw a slope field at the specified points for the differential equation $\dfrac{dy}{dx} = \dfrac{1}{x}$

Sketch a possible solution going through (1, 1).

Evaluate each integral. Use a "u-substitution" and show the new integral based on the function u.

16. $\displaystyle\int_{2}^{3} \dfrac{x}{(x^2 - 3)^2}\, dx$

17. $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (4\sin^2 x \cos x)\, dx$

18. Find $\dfrac{d}{dx}\displaystyle\int_{-1}^{x} \sqrt{t^3 + 1}\, dt$

19. Find $\dfrac{d}{dx}\displaystyle\int_{x^2}^{4} \dfrac{dt}{t + 5}$

20. The acceleration of a particle is given by the function a(t) = 2t - 3 m/s². Given v(0) = 2

   a) Find the displacement of the particle on the interval t = [0, 2]

   b) Set up but do not solve an integral expression to find the total distance travelled by the particle on the interval t = [0, 2]. Your integral expression may NOT contain an absolute value.

21. Find a value x* on [0, 3] which satisfies the Mean Value Theorem for $f(x) = 4x - x^2$

22. Find the average value of the function $f(x) = x^3 - x + 1$ on the interval from [0, 2].